Relationship between Image Gaze Location and Fractal Dimension

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Abstract—This paper predicts that gaze and fractal dimension of images may offer some of measurability to when humans look at still or moving images expanding before one’s eyes such as pictures or scenery, whether humans are subconsciously regarding an image or vaguely looking at the image overall, and whether there is measurability. Several paintings were used to investigate this prediction, in which the gaze location and gaze stopping time of subjects were measured using an eye tracker camera. The relationship between gaze location and fractal dimension was also investigated. The analysis showed that fractal dimension was higher for those areas where gaze was concentrated than for other areas. Fractal dimension calculations were analyzed by both the planar method and the three-dimensional method, and the same tendency was observed for each method.

I. INTRODUCTION

The reason behind this research on image gaze location and fractal dimension lies in a newspaper article called “Painter Jackson Pollock’s Modern Painting (No. 5) is World’s Most Expensive Painting.” From this arose the basic question of how such an avant-garde painting of questionable aesthetic beauty could fetch such a high price. Pollock’s painting was immediately subjected to a fractal analysis. The painting overall and its components all returned very high fractal dimension scores. Based on the hypothesis that the high fractal dimension scores may trigger a physiological visual attraction in humans, an experiment was conducted using six different paintings to measure gaze location and stopping time using an eye tracking camera. Section 2 describes the experimental method used to investigate gaze location and stopping time using an eye tracking camera, and painting preference. In Section 3 using the results of Section 2, a rectangular section is extracted from areas that had highest gaze scores based on observations of the first five gaze stopping locations, a fractal analysis of those areas is performed, several other rectangular areas of the same size are extracted and fractal analyzed, and the fractal dimension averages of those areas are compared with the rectangular sections that had the highest gaze. From the results, a correlation between gaze and fractal dimension was observed. Section 4 describes the fractal dimension calculation method and theory. Section 5 discusses the results as well as the future potential applications of this method.

II. METHOD

A. Participants

Three adults (27–35 years old) participated in the experiment. They were naïve to the purpose of the experiment and had normal or corrected-to-normal vision. None showed any oculomotor pathology.

B. Apparatus

The experiment was controlled by an Apple Macintosh G4 computer. A 21-inch cathode-ray tube (CRT) monitor was used for stimulus presentation. The participant sat on a chair with the head stabilized by a headrest and a chin rest. The CRT was placed in front of the participant at a distance of 80 cm. An Eyelink II eye tracker (SR Research Ltd.) was used for recording eye movements. The sampling rate of the eye tracker was 250 Hz. The Psychophysics toolbox (Brainard, 1997; Pelli, 1997) and the Eyelink toolbox (Cornelissen, Peters, & Palmer, 2002) were used for stimulus presentation and controlling the eye tracker. The keyboard connected to the computer was located in front of the participant for manual responses.

C. Materials and Procedure

The eye tracker was calibrated and the data validated at the beginning of an experiment for each participant, using the Calibration/Validation procedures of Eyelink II. After instruction, participants performed four trials. In each trial, a painting was presented on the screen for 10 sec and the eye movements of participants were recorded. Four paintings for depicting persons and landscapes were used. The order of the paintings was counterbalanced across participants. After each painting presentation they were asked to rate whether he/she liked or disliked it with a seven-point scale. After all four trials, they were given printed versions of the paintings used in the experiment and were required to mark the first three spots where they thought the most attractive or impressive locus in each painting.

III. RESULTS

The rating of like-dislike is not a main task and it is just for participants to actively watch paintings. The analysis focused on the relationship between the gaze location and the fractal coefficient. Figure 1 shows the gaze locations of each participant: Different symbols represent each participant and the number 1, 2, and 3 represent the first, second, and third gaze locations respectively.
A. Results Analysis

In order to present images to subjects and investigate what points they looked at on the images, gaze location coordinate and stopping time was measured for the first through fifth gaze location. There were six different paintings and five subjects. Figure 1 shown according to the artist’s name, the paintings used were Painting 1. Leonardo da Vinci (Mona Lisa), Painting 2. Alfons Maria Mucha, Painting 3. Johannes Vermeer, Painting 4. Jean-Baptiste Camille Corot, Painting 5. Francois Boucher, and Painting 6. Giorgio Morandi.

B. Gaze and Movement Analysis

Gaze coordinates 1 through 5 were obtained and the distribution was analyzed. Paintings were sized to equal the vertical height of a 480 x 640 pixel field; horizontal gaps were filled with a white blank background. Gaze data indicates where subjects looked within the 480 x 640 pixel field, regardless of the actual picture dimensions. Examples are shown in Figure 2. The red markers are gaze coordinates.

C. Gaze Stopping Time Distribution for each Painting

Figure 4 shows the gaze location stopping time distribution for each painting. Stopping time is increased for the second and third gaze locations. The figure also shows that the second gaze stopping time is longest for Painting 1 and Painting 2.

D. Likeability Analysis for each Painting

Figure 5 shows the distribution of likeability subjects showed for each painting.

From the graph, we can see that Painting 3, Vermeer, was the most likeable.
E. Relationship between Fractal Dimension and Gaze

Rectangular areas where gaze was concentrated as described in 3.1 were extracted from each painting and the fractal dimensions were calculated. Two types of fractal dimension calculations – the planar method and the three-dimensional method – were performed. Another rectangular area of the same size was also extracted from the same painting and fractal dimension was calculated. The extracted areas for paintings 2 and 4 are shown in yellow rectangles in Figure 6 and Figure 7.

F. Fractal Dimension Calculation Results

After the fractal dimension value for the area where gaze was concentrated as described in the method 3.3 and three or more other areas were selected and fractal dimensions calculated, the averages were obtained and compared. Figure 8 shows the fractal dimension relationship for each painting. We can see that those areas where gaze was concentrated had higher fractal dimensions than other areas. This same tendency was seen for both the planar method and the three-dimensional method.
IV. FRACTAL DIMENSION CALCULATION METHOD

The word fractal, introduced by Mandelbrot was used to describe the irregular structure of many natural objects and phenomena. The fractal geometry is that nature exhibits a fundamental character generally known as self-similarity. That means, however a complex the shape and/or dynamic behavior of a system, by observing it carefully and imaginatively enough, one can find features in one scale which resemble those at other scales.

The fractal model of an imaged 3D surface, including one of digital photo imagery, furnishes a natural description of most textured and shading images. Defining characteristics of a fractal is that it has a fractal dimension. A fractal dimension of an image grey level intensity surface corresponds quite closely to our intuitive notion or roughness. An approach for inferring fractal dimension of the 3D surface from the image data is assumed that the grey level \( I(x) \) at pixel \( x \) is changed according to a fractal Brownian function (Pentland 1984). After some simplification operations, it gives a relation of

\[
E[I(x + \Delta x) - I(x)]^{H/\Delta x} = C
\]

for any displacement of \( \Delta x \) in pixels within an image. \( E[\cdot] \) is the mean value when keeping \( \Delta x \) (pixel) fixed. H and C are constants. If taking logarithm operations on equation (1), we obtain value of H as a slope of regression line fit to log-log data. The fractal dimension \( D \) of the image surface is derived from

\[
D = 3 - H
\]

Furthermore in order to simplify the calculation, there were two methods developed to deal with easily and quantitatively the imagery pictures (Shimada et al. 2000). One of the methods, called cubic method, is an approach for inferring fractal dimension of the 3D intensity surface from an image data concerning covering processes in increased resolutions [2]. Considering a cube with each edge of size \( r \) pixels, if the number required covering the surface of image with such cubes is \( N(r) \), and further if there exists a relationship as

\[
N(r) - r^H = C
\]

with a constant \( C \), then H gives an estimation of fractal dimension of the image surface.

Figure 9 shows an area \( A \) on which an image surface is assumed. Considering a volume unit of cube in size of \( r \times r \times r \) and its area unit of \( r \times r \) on \( A \), if the surface portion above the area of \( r \times r \) is completely covered by the cubes, the required number \( n(r) \) is

\[
n(r) = \text{floor} \left( \frac{\max(f_i) - \min(f_i)}{r} \right) + 1
\]

Finally, fractal dimension \( D \) is estimated by the regression fit to successive log-log data points of \( \log_{10}(r) \) vs. \( \log_{10}N(r) \), with increased sizes of \( r \).

Another approach, called area method, is concerning count covering numbers with area instead of cubic unit. This calculates the fractal dimension representing picture density undulating complexity calculated using a method to change the coarse graining degree and parameter-based model method.

\[
H = \frac{-\log_{10} \left( \frac{E[F(at)]}{E[F(t)]} \right)}{\log_{10} \left( \frac{1}{a} \right)}
\]

Considering equation (6), setting \( t \) to a fixed value and calculating the number of partitions as a calculation of the length unit of \( E[F(a)] \) as \( 1/a \), or calculating the surface number of partitions for the image density curved surface as the minimum area unit of \( \frac{1}{a} \times \frac{1}{a} \), the above equation can be represented hereafter as \( N(1/a) \). By setting \( 1/a = r \) here, equation (1) can be rewritten as

\[
\log_{10} N(r) = -H \log_{10} r + \log_{10} N(1)
\]

For images, the calculated value on the left side of equation (2) is dependent upon \( r \), so variability is normal. However, if the image is fractal in nature, linearity should be largely preserved.
Thinking in this way, fractal dimension $H$ can be calculated by determining the slope of the regression line from the method of least squares for multiple sample values $(\log_{10} r, \log_{10} N(r)) \cdot (r = 1, 2, \cdots)$ and using this as the estimated value for $H$. For two-dimensional objects such as images, setting the image density curved surface area, calculated using units of small areas such as $r \times r$, to $S(r)$, we obtain:

$$S(r) = r^2 \cdot N(r) \tag{8}$$

Therefore, equation (2) becomes can be written as:

$$\log_{10} S(r) = (2 - H) \log_{10} r + \log_{10} S(1) \tag{9}$$

Here, the surface area of $r \times r$ small area density curve is a triangular area with twice the surface area as shown in Figure 1 of $(i, j, f(i, j)), (i + r, j, f(i + r, j)), (i, j + r, f(i, j + r))$.

By deriving the regression line slope from the method of least squares from multiple sample values of $(\log_{10} r, \log_{10} S(r)) \cdot (r = 1, 2, \cdots)$, the estimated value $2 - H$ can be used to calculate fractal dimension $H$.

V. DISCUSSION

This study showed that subject gaze was concentrated on high fractal dimension areas for all paintings observed. The reasons for this result could be the subject of a future study. Previous research has shown that the human eye exhibits a chaos-like behavior when gathering information. It is highly possible that there is a resonance effect between paintings with high fractal dimensions and eyes engaging in chaotic behavior. It is certainly possible that, like preferences for music with great undulations, or the tendency for communication between two individuals to become more active when there is great undulation in the Lyapunov index due to pulse waves, the same relationship may be occurring between paintings and vision. However, more data is needed for further investigation. Age, cultural background and nurture are also thought to play large roles.

VI. CONCLUSION

We shall continue to conduct experiments to gather more data with increased numbers of subjects and pictures. There also stands the issue of painting likeability, which highlights the necessity for experiments that separate pleasant and unpleasant pictures or that take cultural background into consideration. Additionally, not limited to only paintings, the findings of this research may be applicable to images in various other fields such as calligraphy, natural scenery, graphic design imagery found on websites or advertising, or placement of signage calling for safety. We shall continue to perform various experiments and research.

REFERENCES